Lecture 6

- Review
- Boltzmann equation (cont.)
- Freeze in and Freeze out
- Decoupling of photons

Boltzmann equation (collision integral)

- We would like to include the interactions among particles. We will focus on $2 \to 2$ scattering (collisions) that can transfer momentum and hence change the distribution.

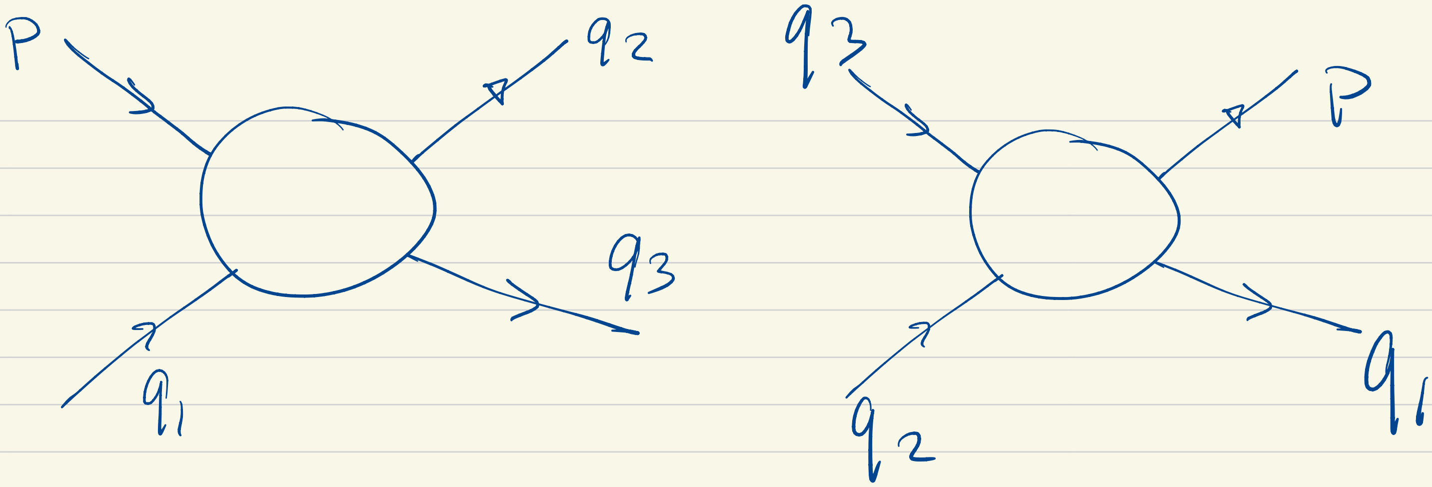
$$\frac{\partial n}{\partial t} - Hp \frac{\partial n}{\partial p} = I_{col.}$$

[This is the Boltzmann equation]

$$I_{col.} = -\frac{1}{2p^0} \int \frac{d^3q_1\, d^3q_2\, d^3q_3}{(2\pi)^9\, 2q_1^0\, 2q_2^0\, 2q_3^0} (2\pi)^4 \cdot$$

$$\cdot\, \delta(p+q_1-q_2-q_3)\, |M_{fi}|^2 \cdot n(p)\, n(q_1) \cdot$$

$$\Big([1 \pm n(q_2)][1 \pm n(q_3)] - n(q_3)\, n(q_2) \cdot [1 \pm n(p)][1 \pm n(q)] \Big)$$



It is in general hard to solve, so we will use approximations.

- Relaxation-time approximation:

$n \sim n_{eq}$.

if $H=0$, $\left.\frac{\partial n}{\partial t}\right|_{n_{eq}} = 0 \iff I_{coll}(n_{eq}) = 0$

can show for Fermi–Bose

$I_{col} \to -\Gamma(n - n_{eq})$

Γ = rate of reaction

$$\Gamma = \tau^{-1} = \left(\left\langle \frac{\lambda}{v} \right\rangle\right)^{-1} = \langle \sigma N v \rangle$$

$v \to$ relative velocity, $\lambda = (\sigma n)^{-1}$ is the mean free path, σ – cross-section.

It is the "total cross section" which uses equilibrium values of distributions of momenta and number of particles.

- We can also integrate the Boltzmann eq. $\int d^3 p$ to get

$$\frac{dN}{dt} + 3HN = -\Gamma (N - N_{eq})$$

Freeze in and freeze out

- Our Boltzmann equation has two terms: one which drives the theory away from thermal state and one that drives it towards it

$\to$ both terms depend on temperature:

σ is a function of v and $\langle v \rangle$ depends on T

H is a function of time, and T is related to t.

→ There are two asymptotic regimes $\Gamma(T) \gg H(T)$ and $\Gamma(T) \ll H(T)$:

$$N \begin{cases} \simeq N_{eq}(T) & , \Gamma \gg H \\ \sim N_{eq}(T_*) \dfrac{a(T_*)^3}{a(T)^3} & , \Gamma \ll H \end{cases}$$

Here T_* is the temperature at which two effects are equal.

[note an approximation in the second term]

→ T_* is called the Freeze out or Freeze in themperature depending on whether $\frac{\Gamma}{H}$ is decreasing or increasing.

[In the very late universe things freeze out eventually]

Electron- Positron gas

Let us work out a concrete example: e^+ and e^- thermolizing with γ's during radiation domination ($a \sim t^{1/2}$)

Cross-sections can be computed using Quantum Electro Dynamics. The dominant process is

$$e^+ e^- \longleftrightarrow \gamma\gamma$$

$$\sigma = \begin{cases} \frac{1}{2v} \pi r_e^2 & v \ll 1 \\ \frac{m_e^2}{E^2} \pi r_e^2 \left(\log \frac{4E^2}{m_e^2} - 1 \right) & v \approx 1 \end{cases}$$

$$r_e = \frac{\alpha}{m_e} \;, \quad \alpha = \frac{1}{137} \qquad (\alpha \sim q^2)$$

E = center of mass energy.

$T \lesssim m_e \rightarrow v \ll 1$ non-relativistic

$T \gtrsim m_e \rightarrow v \sim 1$ relativistic ($E \sim T$)

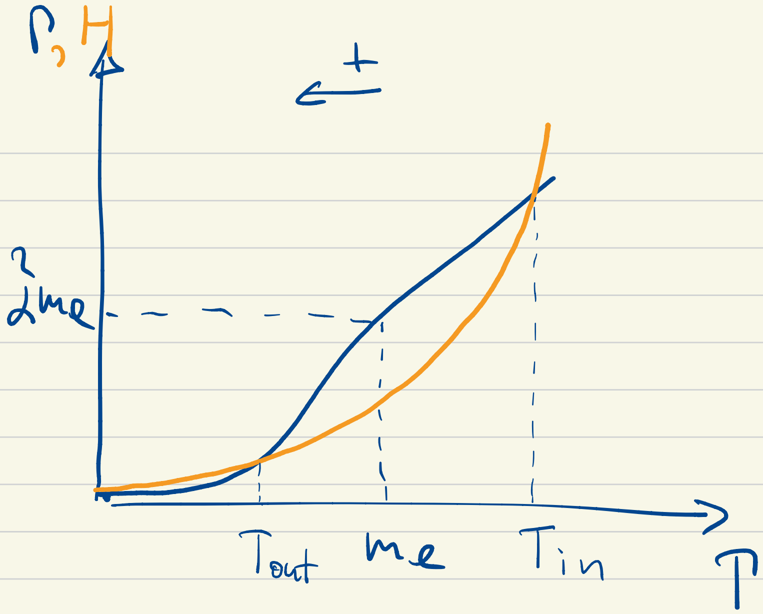
$$n_i = g_i \left(\frac{m_i T}{2\pi}\right)^{3/2} e^{\frac{\mu_i - m_i}{T}}$$

$$n_i = \frac{3}{4} \frac{\zeta(3)}{\pi^2} g_i T^3$$

$$\Gamma = \langle \sigma n v \rangle = \begin{cases} \sim r_e^2 (m_e T)^{3/2} e^{-\frac{m_e}{T}} \\ \\ \sim \alpha^2 T \end{cases}$$

where we ignored the log in the cross section as well as order-one constants

[at $T \sim m_e$ both give $\alpha^2 m_e$]



H(T) in radiation domination is given by $\frac{T^2}{M_0}$ (see lect. 5, $M_0 \sim M_{pl}$)

$$T_{in}: \quad \alpha^2 T = \frac{T^2}{M_0} \Rightarrow T_{in} = \alpha^2 M_0 \sim$$

$$\sim 10^{14}\,\text{GeV}$$

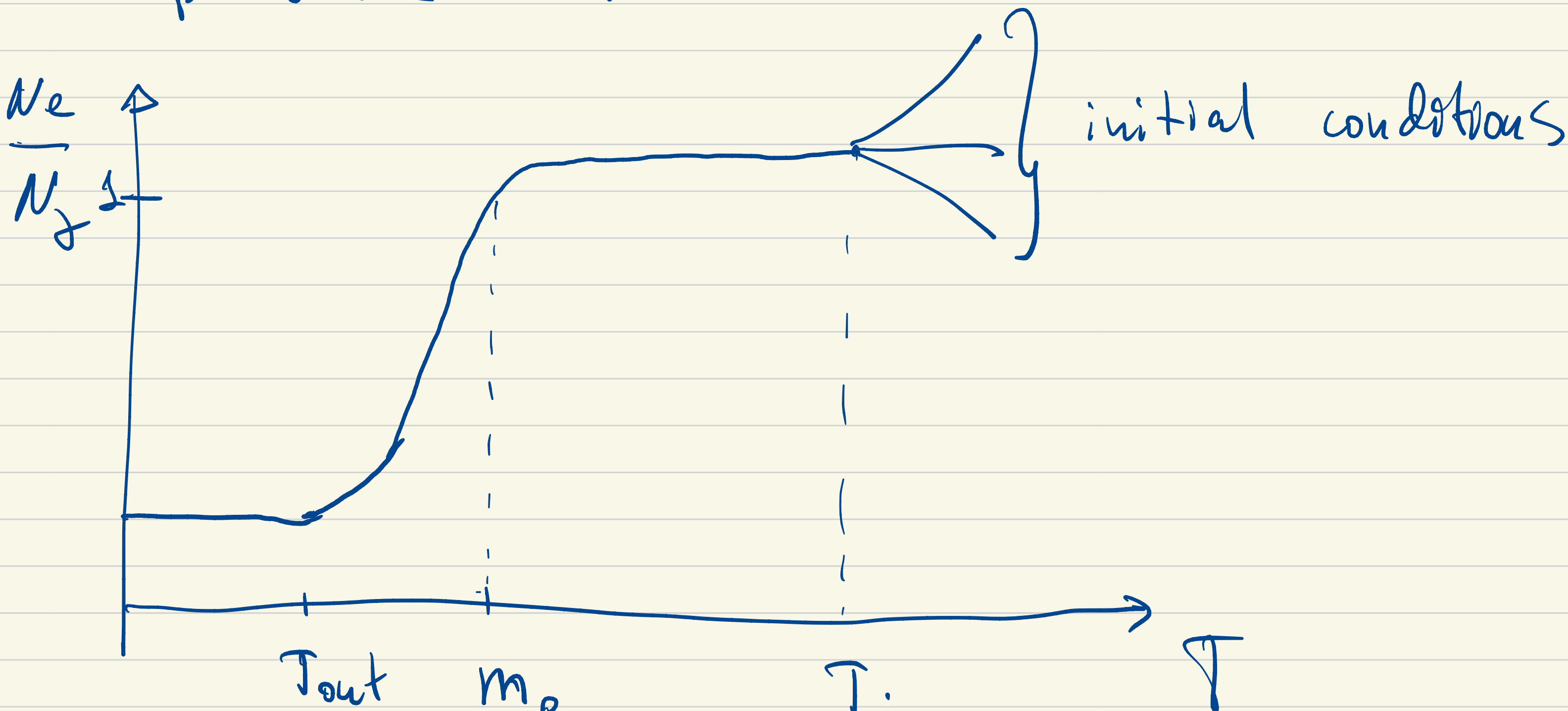
$$T_{out}: \quad r_e^2 (m_e T)^{3/2} e^{-\frac{m_e}{T}} \frac{M_0}{T} = 1$$

introduce $x = \frac{m_e}{T_{out}}$:

$$e^{-x} x^{\frac{1}{2}} \simeq \frac{m_e}{\alpha^2 M_0} \simeq \frac{10^6}{10^{-4} \cdot 10^{19} \cdot 10^9} \approx 10^{-18}$$

$$x \simeq 40 \rightarrow T_{out} \simeq \frac{m_e}{40} \simeq 10\ \text{KeV}$$

We can assume that photons are in thermal equilibrium, their number density scales as $n_\gamma = a^{-3}$. This is consistent with the fact that their distribution stays thermal, just the temperature varies



Most important is that final particle density of e^+ and e^- is independent of the initial conditions!